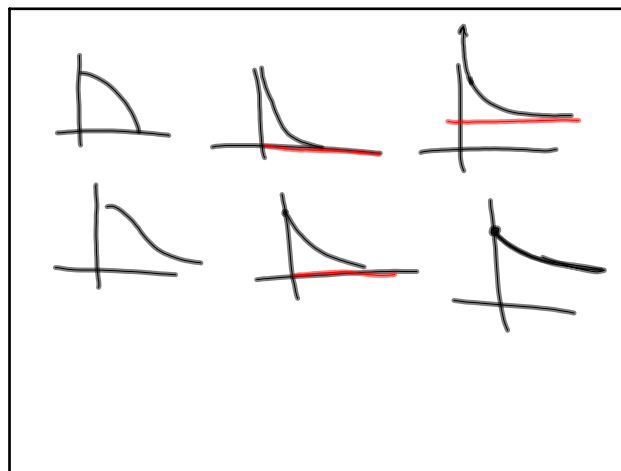


6.4b Exponential Growth and Decay

Newton's Law of Cooling: The rate at which an object's temperature is changing is directly proportional to the difference between its temperature and the temperature of the surrounding medium.

$T = \text{Temp}$
 $\frac{dy}{dt} = k \cdot y$
 $y = T - T_s$
 $T = \text{Temp}$
 $T_s = \text{Temp of surrounding}$
 $y = y_0 e^{kt}$
 $T - T_s = (T_0 - T_s) e^{kt}$
 $T_0 = \text{initial temp}$
 $T = (T_0 - T_s) e^{kt} + T_s$



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Dec 9-9:32 AM

A hard boiled egg at 98 degrees Celsius is put in a pan under running 18 degree water to cool. After 5 minutes, the egg's temperature is found to be 38 degrees. How much longer will it take the egg to reach 20 degrees?

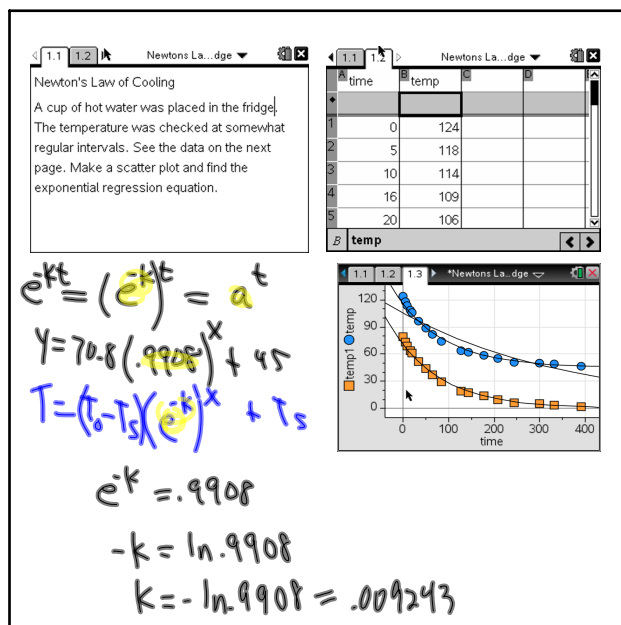
$$38 = (98 - 18) e^{-k \cdot 5} + 18$$

$$k = .277$$

$$20 = (98 - 18) e^{-.277t} + 18$$

$$t = 13.3 \text{ total}$$

$$\frac{-5}{8.3} \text{ longer}$$



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Separation of Variables

Solve for y if $\frac{dy}{dx} = (xy)^2$ and $y=1$ when $x=1$ *initial condition*

$$\frac{dy}{dx} = x^2 y^2$$

$$\text{sep. var. } \int \frac{dy}{y^2} = \int x^2 dx$$

$$\int y^{-2} dy = \frac{x^3}{3} + C$$

$$-y^{-1} = \frac{x^3}{3} + C$$

$$-\frac{1}{y} = \frac{x^3}{3} + C$$

$$\frac{1}{y} = -\frac{x^3}{3} + C$$

$$1 = \frac{1}{-\frac{1}{3} + C}$$

$$C = \frac{4}{3}$$

$$y = \frac{1}{-\frac{x^3}{3} + C}$$

$$x=1 \quad y=1$$

$$y = \frac{1}{-\frac{x^3}{3} + \frac{4}{3}} = \frac{1}{-\frac{x^3-4}{3}} = \frac{3}{4-x^3}$$

Dec 6-10:49 PM